Int. J. Heat Mass Transfer. Vol. 19, pp. 821–826. Pergamon Press 1976. Printed in Great Britain

THE EFFECT OF GAS HEAT STORAGE UPON THE PERFORMANCE OF THE THERMAL REGENERATOR

A. J. WILLMOTT* and CLARE HINCHCLIFFE
Department of Computer Science, University of York, Heslington, York, United Kingdom

(Received 29 April 1975)

Abstract—Most mathematical models of a thermal regenerator incorporate the assumption that the residual gas in the regenerator packing is ejected at the end of a period of operation and the regenerator is refilled with new gas before the next period begins. It is further assumed that this idealised evacuating and refilling of the channels of the packing at each reversal causes no change in the packing temperature.

This paper describes a model of the regenerator in which a representation of gas carryover is attempted. The effect of the carryover predicted by this model is presented.

NOMENCLATURE

- A, heating surface area of regenerator packing [m^2];
- C, specific heat of matrix [kJ/kg°C];
- h, heat-transfer coefficient between gas and solid [W/m^2 °C];
- L, length of regenerator [m];
- M, mass of packing [kg];
- m, mass of gas resident in channels of packing [kg];
- P, length of period [s];
- S, isobaric specific heat of gas [kJ/kg°C];
- T, temperature of packing [°C];
- t, gas temperature [°C];
- thi, hot period inlet gas temperature [°C];
- tci, cold period inlet gas temperature [°C];
- thx, hot period exit gas temperature [°C];
- tcx, cold period exit gas temperature [°C];
- thm, chronological mean exit gas temperature, hot period [°C];
- tcm, chronological mean exit gas temperature, cold period [°C];
- W, mass flow rate of gas [kg/s];
- y, distance from entrance down the length of regenerator [m].

Greek symbols

- ξ, dimensionless distance;
- η, dimensionless time;
- $\eta_{REG}^{\delta\Pi}$, thermal ratio calculated with effect of carry over incorporated [dimensionless];
- η_{REG}, thermal ratio, carryover ignored [dimensionless];
- Λ, reduced length [dimensionless];
- Π, reduced period [dimensionless];
- μ, ratio M/m [dimensionless];
- σ, ratio C/S [dimensionless];
- θ, time [s].

Superscripts

- $'$, refers to hot period;
- $''$, refers to cold period.

INTRODUCTION

THE BEHAVIOUR of a regenerator can be idealised by a model in which a gas enters at constant temperature at one end of a heat storing packing, loses or retrieves heat as it passes through the channels of the packing before departing with a varying exit temperature. This gas is then shut off at the end of the period and in most models it is assumed that all the residual gas is ejected from the matrix channels. In this paper it is assumed that the residual gas is pushed out by the other gas which enters at the opposite end of the packing at the start of the next period. No mixing is assumed to occur between the residual and the incoming gases. The effect of finite reversal time is neglected. After a sufficiently large number of cycles, the temperature behaviour of the regenerator becomes periodic.

This idealisation also embodies further assumptions specified by Willmott [1] which imply the simplest linear model of the regenerator, for which, for the case where the effect of the carryover is ignored, several methods of solution have been proposed (for example, Hausen [2], Iliffe [3], Willmott [4] and Nahavandi and Weinstein [5]). However, the purpose of this paper is to attempt an estimate of the possible effect of gas carryover at the regenerator reversals upon thermal effectiveness. Although it is idealised that the gas left in the regenerator does not mix with the new incoming gas at the start of the next period, in practice some mixing will occur. These assumptions may seem fairly restrictive in present day research and computing capability. However, the model should predict the circumstances under which the effect of gas carryover might be important and should be included within more complicated non-linear models.

*Present address: Division of Mechanical Engineering, Commonwealth Scientific and Industrial Research Organisation, Highett, Victoria, Australia.

MATHEMATICAL MODEL

The temperature behaviour of the gas and solid packing can be represented by the differential equations

$$\frac{\partial t}{\partial \xi} = T - t, \tag{1}$$

$$\frac{\partial T}{\partial \eta} = t - T. \tag{2}$$

These equations were established by Hausen [6] and Nusselt [7]. The dimensionless measures of distance ξ and time η are defined:

$$\xi = \frac{hA}{WSL}y, \tag{3}$$

$$\eta = \frac{hA}{MC}\left(\theta - \frac{m}{WL}y\right). \tag{4}$$

In these equations one or two primes are inserted against the various symbols corresponding to the hot or cold period respectively. Hausen [6] proposed a pair of dimensionless parameters for each period of operation namely

$$\Lambda = \frac{hA}{WS} \quad \text{(Reduced length)}, \tag{5}$$

$$\Pi = \frac{hA}{MC}\left(P - \frac{m}{W}\right) \quad \text{(Reduced period)}. \tag{6}$$

The term my/WL can be called the "gas heat storage term" and represents the effect of the heat capacity of the gas resident in the regenerator at any instant. This is illustrated in Fig. 1.

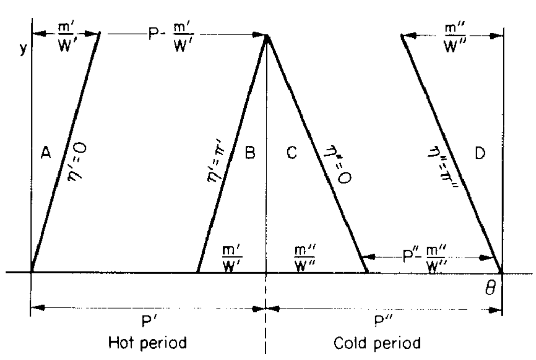

FIG. 1. Representation by model of regenerator behaviour. Triangles A and C represent the ejection of gas left resident in the packing at the end of the previous period. Triangles B and D represent the accumulation of gas in the regenerator packing at the end of the hot and cold periods respectively.

Integration along the lines corresponding to constant values of η, for example $\eta' = 0$ or $\eta'' = \Pi''$, represents the following of the temperature history of a gas molecule as it moves down the regenerator. Thus the gas heat storage term causes a time delay, which increases along the length of the regenerator, in the manifestation of changes in both solid and gas temperature predicted using a zero gas heat storage ($m = 0$) model. Clearly, the greater the value of the flow rate W, the sooner the influence of the gas entering the regenerator will be felt, and thus the smaller the gas heat storage term.

For the reversing regenerator, the triangles marked A, B, C and D on Fig. 1 may become important. A is the time–space area representing the expulsion of gas which enters but does not leave the regenerator in the time–space area D. Similarly triangle C represents the expulsion of gas which enters but does not leave the regenerator in the time–space area B.

The reversal boundary conditions embody the assumption that the gas and solid temperatures at the end of the hot/cold period are equal to those at the beginning of the cold/hot period, and are written as:

$$T'(y, P') = T''(L-y, 0), \; t'(y, P') = t''(L-y, 0), \tag{7}$$

$$T''(y, P'') = T'(L-y, 0), \; t''(y, P'') = t'(L-y, 0) \tag{8}$$

or in terms of the dimensionless parameters for the solid temperatures:

$$T'(\xi', \Pi' + \xi'/\mu'\sigma') = T''(\Lambda''(1-\xi'/\Lambda'), \; -\Lambda''(1-\xi'/\Lambda')/\mu''\sigma''), \tag{9}$$

$$T''(\xi'', \Pi'' + \xi''/\mu''\sigma'') = T'(\Lambda'(1-\xi''/\Lambda''), \; -\Lambda'(1-\xi''/\Lambda'')/\mu'\sigma'). \tag{10}$$

In the hot period m'/w' is the time(s) and $\Lambda'/\mu'\sigma'$ is the corresponding reduced time for the regenerator to be expelled of all the gas left in the regenerator at the end of the previous cold period. m'/w' is also the time during which gas enters but does not leave the regenerator at the end of the hot period. It is convenient to define $\delta\Pi'$ and $\delta\Pi''$ by

$$\delta\Pi' = \Lambda'/\mu'\sigma', \tag{11}$$

$$\delta\Pi'' = \Lambda''/\mu''\sigma''. \tag{12}$$

The differential equations (1) and (2) have been solved by the finite difference method described by Willmott [4], expanded to facilitate integrations over the triangles A, B, C and D, Fig. 1.

In the time–space triangle representing the expulsion of gas at the start of the period (see Fig. 2), the number of space and time steps are set equal where

$$\Delta\xi = \Lambda/m,$$

$$\Delta\eta = \delta\Pi/m.$$

Along the line AB, that is at positions $(r, m-r)$, $r = 0, 1, 2, \ldots, m$, the initial conditions are prescribed by the gas and solid temperatures at the end of the previous period. The gas at position $m-1$ moves to the exit in time $\Delta\eta$. The expulsion of this gas effects a change in the solid temperature at the exit and the modified temperature is computed together with the temperature of this gas at the exit. Meanwhile the gas at position $m-k$ moves to $m-k+1$ $(k = 2, 3, \ldots, m)$ and the effect upon solid and gas temperatures is calculated. The process is repeated over the remaining time as this carryover gas leaves the regenerator. The simulation of the main part of regenerator operation then proceeds.

At times greater than $\eta > \Pi$, the regenerator admits

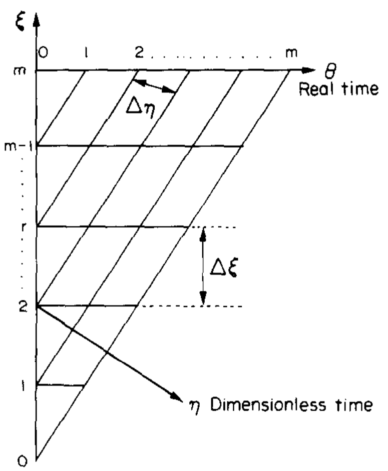

FIG. 2. Space-time grid for finite difference solution of partial differential equations for ejection of residual gas at the start of a period.

gas which does not leave the channels of the packing. The temperature behaviour of both gas and solid as this gas accumulates in the packing is calculated in an exactly analogous manner to that employed for the expulsion of gas at the start of the period.

This procedure is applied to the next period of the cycle. The regenerator is simulated in this way until cyclic equilibrium is attained.

EFFECT OF GAS CARRYOVER UPON THERMAL PERFORMANCE

Two measures of the effectiveness of regenerator performance are given by the thermal ratios η'_{REG}, η''_{REG}. Consideration here is restricted to the symmetric case where $\Lambda = \Lambda' = \Lambda''$ and $\Pi = \Pi' = \Pi''$ in which case the two thermal ratios are the same and equal to η_{REG}.

If $thx\,(\theta)$ and $tcx\,(\theta)$ are the time varying exit gas temperatures in the hot and cold periods respectively, and thm and tcm are their chronological means:

$$thm = \frac{1}{P'}\int_0^{P'} thx(\theta)\,\mathrm{d}\theta,\quad tcm = \frac{1}{P''}\int_0^{P''} tcx(\theta)\,\mathrm{d}\theta,$$

then

$$\eta'_{REG} = \frac{thi - thm}{thi - tci}$$

$$\eta''_{REG} = \frac{tcm - tci}{thi - tci}.$$

In the model in which gas left in the regenerator is driven out by the incoming gas, $thx\,(\theta)$ records initially the exit temperature of this residual gas as it is expelled. The observed thermal ratio is therefore modified and we have elected here to measure the effect of gas carryover by the ratio of the thermal ratio $\eta^{\delta\Pi}_{REG}$ where the gas carryover is incorporated into the model, to the thermal ratio η_{REG} computed when this effect is ignored. For the symmetric case $\eta^{\delta\Pi}_{REG}$ is the same for both hot and cold periods.

The ratio $\eta^{\delta\Pi}_{REG}/\eta_{REG}$ has been computed over the range $0 \leqslant \delta\Pi/\Pi \leqslant 0.4$ for $0.33 \leqslant \Lambda \leqslant 9.00$ and for $\Pi = 1$, 2 and 3. The ratios are displayed in Figs. 3(a), 3(b) and 3(c).

INTERPRETATION OF THE EFFECT OF CARRYOVER

The greater the proportion of each of the periods required to expel the residual gas at the start of each of these periods, the more marked is the effect of carryover.

At the end of a period gas enters the regenerator with constant inlet temperature but is prevented from leaving the regenerator by the occurrences of a reversal. This same gas is driven out of the regenerator in the opposite direction, losing or recovering some of the heat it gained or lost in the packing when it was admitted initially. This gas leaves the regenerator at a temperature approaching that with which it entered. Where the exit gas temperature for the remainder of the period, once the residual gas has been expelled, also approaches the inlet gas temperature of the previous period (high value of thermal ratio), the reversals have little effect upon regenerator behaviour. For Λ greater than about 3.5, for the values of Π we have considered, a slight deterioration of thermal ratio is observed, and this deterioration increases with reduced length.

For small values of reduced length, the thermal ratio is relatively small. The expulsion of hot gas at the start of the cold period, for example, significantly improves the thermal ratio, since a relatively cool exit temperature during the main part of the period of operation must be chronologically averaged with the expelled hotter gas to yield a higher thermal ratio $\eta^{\delta\Pi}_{REG}$; indeed for $\Lambda < 3$, $\eta^{\delta\Pi}_{REG}$ increases very rapidly by between 2% and 18% for small changes of $\delta\Pi/\Pi$ from 0 to 2%. $\eta^{\delta\Pi}_{REG}$ continues to rise, but less rapidly for $\delta\Pi/\Pi > 2\%$, the rate of increase of $\eta^{\delta\Pi}_{REG}$ rising with decreasing reduced length.

Granville *et al.* [8] attempted a comparison between experimental thermal ratios of packed regenerators and thermal ratios predicted by models which excluded the effect of the gas carryover. They concluded that for $\Lambda/\Pi > 2$, good agreement between experiment and prediction is likely but that for $\Lambda/\Pi < 2$ actual thermal ratios obtained in practice may be greater than those predicted theoretically. Granville *et al.* interpreted this discrepancy in terms of the thermal capacity of the walls of the regenerator, and specified that the reduced length Λ should be modified. However the heat capacity of a regenerative system is usually incorporated within the dimensionless parameter reduced period Π, which they did not modify.

In Fig. 4 is reproduced the graphical presentation of the experimental results set out by Granville *et al.* Our model predicts this same deterioration in the correlation between the thermal ratio predicted with and without the effect of carryover being considered for $\Lambda/\Pi < 2$. However Granville *et al.* found that in all cases, the predicted thermal ratio was less than that observed experimentally, whereas our model predicts thermal ratios greater than that computed using a conventional model in some cases, smaller in others.

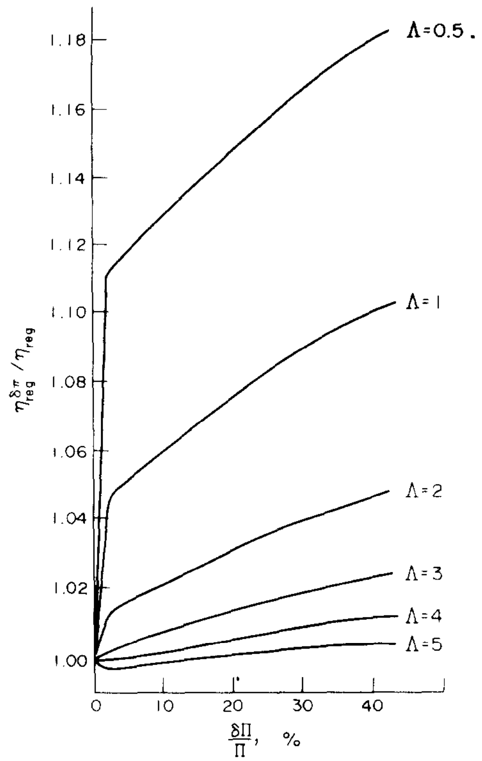

FIG. 3(a). Effect of carry over upon thermal ratio ($\Pi = 1$).

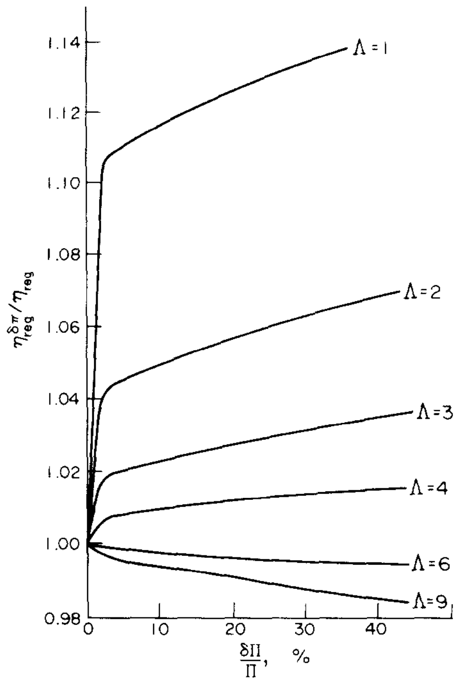

FIG. 3(c). Effect of carry over upon thermal ratio ($\Pi = 3$).

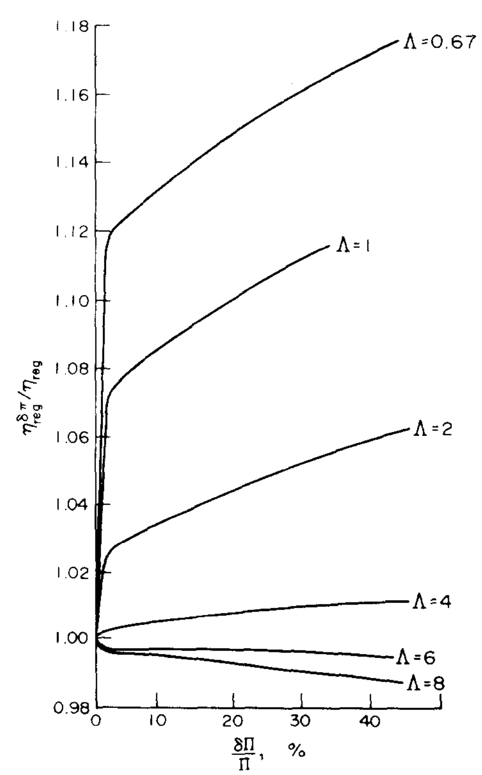

FIG. 3(b). Effect of carry over upon thermal ratio ($\Pi = 2$).

Banks and Ellul [9] predicted the effects of by-pass flows upon the performance of rotary regenerators using a simple heat balance method. They considered the general case where carryover from one side of the regenerator to the other could be caused by leakage but not by the residual gas in the regenerator being ejected at the start of the period. Nevertheless their conclusions for leakage effects coincided with ours for carryover, for low values of thermal ratio η_{REG}, and implicitly for low values of Λ/Π.

CONCLUSIONS

Traditional models of the thermal regenerator ignore the effect of gas carryover and this assumption is certainly justified for ranges of operation where $\Lambda/\Pi > 2$ and $\delta\Pi/\Pi < 40\%$. Our calculations suggest at worst a 2–3% deterioration in thermal ratio caused by the carryover effects, and in practice, this deterioration will be less since mixing of the resident and incoming gases is likely to occur.

However, in the case where $\Lambda/\Pi < 2$, great care must be exercised. The inclusion of carryover for $\delta\Pi/\Pi$ as small as 2% will cause a thermal ratio to be predicted which is significantly larger, by as much as 20%, than that computed using a conventional model.

In terms of the rotary regenerator, the results presented here are surprising in that it has been supposed the reversal or carryover effects will become significant at high rotational speeds. The reverse proves to be the case. At low rotational speeds, the low cold side thermal

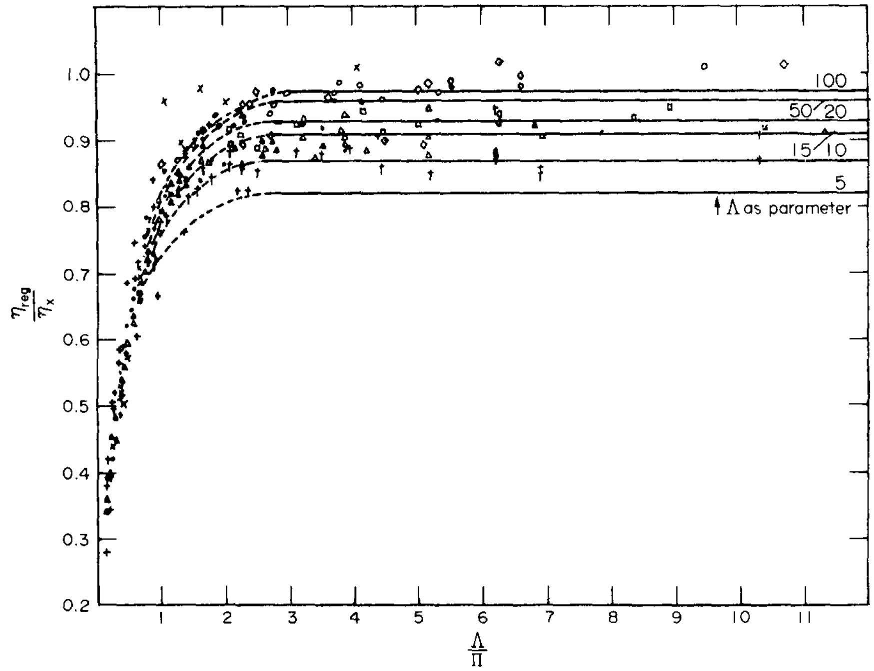

FIG. 4. Importance of ratio Λ/Π on the ratio η_{REG}/η_x. η_x = thermal ratio determined experimentally.

ratio, for example, is enhanced by the hot gas which is carried over at the reversal. High rotational speeds correspond usually to large values of thermal ratio when the effect of carryover is small.

The significance of these results for simultaneous heat and mass transfer in a regenerator remains to be examined. This problem is complicated where the heat and mass transfer are coupled as in the case of a regenerator used for air conditioning where both heat and moisture transfer take place. If the thermal ratio is high but the corresponding ratio for mass transfer is small, our conclusions suggest that carryover effects are likely to be important.

REFERENCES

1. A. J. Willmott and R. J. Thomas, Analysis of the long contra-flow regenerative heat exchanger, *J. Inst. Maths Appls* **14**, 267–280 (1974).
2. H. Hausen, Naherungsverfahren zur Berechnung des Wärmeaustauches in Regeneratoren, *Z. Angew. Math. Mech.* **2**, 105–114 (1931).
3. C. E. Iliffe, Thermal analysis of the counterflow regenerative heat exchanger, *J. Inst. Mech. Engrs* **159**, 362–372 (War Emergency Issue No. 44) (1948).
4. A. J. Willmott, Digital computer simulation of a thermal regenerator, *Int. J. Heat Mass Transfer* **7**, 1291–1302 (1964).
5. A. H. Nahavandi and A. S. Weinstein, A solution to the periodic flow regenerative heat exchanger problem, *Appl. Scient. Res.* **10**, 335–348 (1961).
6. H. Hausen, Über die Theorie des Wärmeaustausches in Regeneratoren, *Z. Angew. Math. Mech.* **3**, 173–200 (1929).
7. W. Nusselt, Die Theorie des Winderhitzers, *Z. Ver. Dt. Ing.* **71**, 85–91 (1927).
8. W. H. Granville, W. I. MacDonald and P. Cutler, Comparison of predicted and experimental thermal efficiencies of packed regenerators, *Proceedings of 3rd International Heat Transfer Conference, Chicago*. Vol. 4, pp. 329–335 (1966).
9. P. J. Banks and W. M. J. Ellul, Predicted effects of by-pass flows on regenerator performance, *Trans. Instn. Engrs Aust.* **MC9**(1 & 2), 10–14 (1973).

EFFET DE LA CHALEUR ACCUMULEE PAR LES GAZ SUR LES PERFORMANCES D'UN REGENERATEUR THERMIQUE

Résumé—La plupart des modèles mathématiques de régénérateurs thermiques retiennent l'hypothèse suivant laquelle les gaz résiduels contenus dans le régénérateur sont éjectés à la fin de la phase d'opération et le régénérateur rempli de gaz frais avant que ne commence la phase suivante. On suppose, en outre, que les opérations idéalisées d'évacuation et de remplacement des gaz dans les canaux de remplissage à chaque phase nouvelle ne modifient pas la température du contenu.

L'article décrit un modèle de régénérateur dans lequel on a essayé de tenir compte du déplacement des gaz. On présente les resultats prévus par le modèle de l'effet du déplacement des gaz.

DER EINFLUSS VON GAS-WÄRME-SPEICHERUNG AUF DAS VERHALTEN VON THERMISCHEN REGENERATOREN

Zusammenfassung—Die meisten mathematischen Modelle eines thermischen Regenerators enthalten die Annahme, daß das Restgas in der Regeneratorpackung am Ende der Arbeitsperiode ausgestoßen und der Regenerator mit neuem Gas gefüllt wird, ehe die nächste Periode beginnt. Es wird weiter angenommen, daß diese idealisierte Evakuierung und Wiederfüllung der Kanäle der Packung bei jeder Umkehr keine Veränderung in der Packungstemperatur hervorruft.

In dieser Arbeit wird ein Regeneratormodell beschrieben, in der eine Berücksichtigung von Restgas versucht wird. Der nach diesem Modell ermittelte Einfluß des Restgases ist wiedergegeben.

ВЛИЯНИЕ АККУМУЛЯЦИИ ТЕПЛА ГАЗА НА ХАРАКТЕРИСТИКУ ТЕРМОГЕНЕРАТОРА

Аннотация — Большинство математических моделей термогенератора основаны на допущении, что остаточный газ в насадке регенератора выбрасывается в конце рабочего цикла и регенератор вновь наполняется новым газом до начала следующего цикла. Далее предполагается, что эта идеализированная откачка и повторное заполнение газом каналов насадки в каждом цикле не вызывает изменения температуры насадки.

Представлена модель регенератора, в которой делается попытка по-новому представить перенос газа. Рассматривается эффект этого переноса, предсказываемой данной моделью.